ПATIBIA UПIVERSITY
FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES

## SCHOOL OF NATURAL AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

| QUALIFICATION: Bachelor of science in Applied Mathematics and Statistics |  |
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| QUALIFICATION CODE: 07BSAM | LEVEL: 6 |
| COURSE CODE: CLS601S | COURSE NAME: CALCULUS 2 |
| SESSION: JUNE 2023 | PAPER: THEORY |
| DURATION: 180 MINUTES | MARKS: 100 |


| FIRST OPPORTUNITY QUESTION PAPER |  |
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| EXAMINERS | MR BENSON OBABUEKI |
|  | DR SERGE NEOSSI-NGUETCHUE |
| MODERATOR: | DR DAVID IIYAMBO |


| INSTRUCTIONS |
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| 1. Answer ALL questions in the booklet provided. |
| 2. Show clearly all the steps used in the calculations. |
| 3. All written work must be done in blue or black ink and sketches must |
| be done in pencil. |

## PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 2 PAGES (excluding this front page)

## Question 1 (35 marks)

Determine the following indefinite integrals using the indicated techniques:
$1.1 \int 3 x e^{x^{2}} d x$ by substitution.
1.2 $\int e^{3 x} \sin 2 x d x$ by parts. Start with $u=\sin 2 x$.
$1.3 \int \frac{d x}{36 x^{2}+1}$ by trigonometric substitution.
1..4 $\int \sin ^{4} y \cos ^{3} y d y$ by any (simple) method.
1.5 $\int \frac{d x}{\sin x}$ using the t-formula.

## Question 2 (14 marks)

2.1 Determine the area enclosed by $y=x^{2}-9$ and $y=9-x^{2}$.
2.2 What is the arc length of $y=\ln \sec x$ in the interval $0 \leq x \leq \frac{\pi}{4}$ ? (Hint: $\sec x$ is positive in the given interval)
2.3 Calculate the volume of the solid generated if $y=2 x$ is rotated about the $x$-axis through a complete revolution, $0 \leq x \leq 4$.

## Question 3 (16 marks)

The definite integral $\int_{0}^{1} e^{x^{2}+3} d x$ is to be estimated using the Simpson Rule, correct to within an error of $0.7 \%$.
3.1 Determine the number of subintervals needed.
3.2 Use $n=8$ to estimate the given definite integral to within an error of $0.7 \%$.

## Question 4 (13 marks)

4.1 Given that $f(x)=\sum_{n=1}^{n=\infty} \frac{n+1}{4^{n+1}} x^{n}$, determine the definite integral $\int_{0}^{1} f(x) d x$.
4.2 Use Taylor series to approximate $\int_{0}^{0.1} \sin \left(x^{4}\right) d x$ to within an error of $10^{-20}$. (Hints: (1) $\sin u=\sum_{n=0}^{\infty} \frac{(-1)^{n} u^{2 n+1}}{(2 n+1)!} ;$ (2) The alternating series error estimation rule allows you to truncate from the first term that has an absolute value less than the error limit.) (7)

## Question 5 (11 marks)

The curve $y=x-x^{2}$, between $x=0$ and $x=2$, rotates about the $x$-axis through a complete revolution. Determine the centre of gravity of the solid so formed.

## Question 6 (11 marks)

A curve is defined by the parametric equations $x=\theta-\sin \theta$ and $y=1-\cos \theta$. Determine the area generated by the curve between $\theta=0$ and $\theta=2 \pi$, when rotated completely about the $x$ -axis.

